

Mathematics Olympiad of Nepal [MOON]

Syllabus for the Second Round Competition (National Level)

1. Arithmetic:
 - A. Time and work
 - B. Profit and loss
 - C. Simple and compound Interest
 - D. Population growth and depreciation
 - E. VAT, discount, commission and tax

2. Mensuration:
 - A. Area and perimeter of rectangle, square, trapezium, triangles etc.
 - B. Area of 4 walls, 4 walls and ceiling, 4 walls, ceiling and floor and area of path
 - C. Cost of carpeting, cementing and coloring etc.
 - D. Lateral surface area, total surface area and volume of triangular prism, pyramids, cube and cuboid
 - E. Curved surface area, Total surface area and volume of cylinder, spheres, hemisphere, cones and combined solids

3. Matrices, determinants and system of linear equations:
 - A. Definition and types of matrices, operation on matrices
 - B. Transpose of matrices, Inverse of a matrices and rank with some properties
 - C. Dependent and independent lines, consistency and inconsistency of the lines
 - D. Solution of the system of linear equations

4. Algebra:
 - A. Laws and indices and simplification
 - B. Polynomial equation and Quadratic equation
 - C. Inequalities
 - D. Diophantine product and cross product of vectors

5. Sequence and series:
 - A. Arithmetic sequence, geometric sequence and harmonic sequence
 - B. Sum to infinity terms and sum to n terms of the series
 - C. Arithmetic geometric series

6. Complex numbers:
 - A. Operation on complex numbers
 - B. Conjugate
 - C. Absolute value
 - D. Square roots, cube roots and its application
 - E. Polar form
 - F. Use of De-Moivers theorem

7. Vectors:
 - A. Vector geometry
 - B. Scalar Operation on complex numbers
 - C. Conjugate and absolute value of a complex numbers
 - D. Square root, cube roots and their properties
 - E. Polar form and De-Moiver's theorem and its application

8. Permutation and combination:
 - A. Basic principle of counting
 - B. Total no of permutations of n distinct objects taken r at a time
 - C. Circular permutation
 - D. Permutations if the objects are repeated
 - E. Permutations if the objects can be repeated
 - F. Combination of the n distinct objects taken r at a time
 - G. Combinations if the objects are repeated
 - H. Mixture problem of Permutation and combination

9. Probability:
 - A. Trials, Events, Mutually exclusive events, Dependent and independent events
 - B. Addition and Multiplication theorem of probability and their applications
 - C. Binomial distribution

10. Set, Relation and Function:
 - A. Method of describing sets and subsets
 - B. Operation on sets and cardinality of sets
 - C. Types of function
 - D. Inverse of a function
 - E. Composite function
 - F. Transcendental function
 - G. Domain and Range
 - H. Symmetricity
 - I. Odd and even function
 - J. Graph of quadratic, Cubic, Trigonometric, Exponential function, Logarithm function

11. Co-Ordinate Geometry:
 - A. Distance formula, Section formula, mid-point formula, Centroid of a triangle, ortho-center, circumventers, slopes and intercepts
 - B. Equation of straight lines in different condition
 - C. Area of triangles and quadrilateral
 - D. Pair of straight lines, homogenous equation of second degree, second degree equation, equation of circle, equation of tangents and normal

12. Geometry:
 - A. Measurement of angle
 - B. Trigonometrical ratios, properties and values
 - C. Inverse circular function
 - D. Properties of triangle
 - E. Compound angles, multiple angles, sub multiple angles and their properties
 - F. Conditional identities, general solution of trigonometrical equation
 - G. Height and distance

13. Theory of Numbers:
 - A. Divisibility and its properties
 - B. Congruence and its properties
 - C. Some important properties on Integers
 - a) Fermat's theorem
 - b) De-Polygnacs theorem

- c) Wilson theorem
- d) Pigeon whole principle
- e) Chinese-Remainder theorem

14. Elementary graph theory:

- A. Complete graph
- B. Regular graph
- C. Bipartite graph
- D. Complete bipartite graph
- E. Sub graph
- F. Induced graph



IMO 2023



Chiba, JAPAN 64th

English (eng), day 1

Saturday, 8. July 2023

Problem 1. Determine all composite integers $n > 1$ that satisfy the following property: if d_1, d_2, \dots, d_k are all the positive divisors of n with $1 = d_1 < d_2 < \dots < d_k = n$, then d_i divides $d_{i+1} + d_{i+2}$ for every $1 \leq i \leq k - 2$.

Problem 2. Let ABC be an acute-angled triangle with $AB < AC$. Let Ω be the circumcircle of ABC . Let S be the midpoint of the arc CB of Ω containing A . The perpendicular from A to BC meets BS at D and meets Ω again at $E \neq A$. The line through D parallel to BC meets line BE at L . Denote the circumcircle of triangle BDL by ω . Let ω meet Ω again at $P \neq B$. Prove that the line tangent to ω at P meets line BS on the internal angle bisector of $\angle BAC$.

Problem 3. For each integer $k \geq 2$, determine all infinite sequences of positive integers a_1, a_2, \dots for which there exists a polynomial P of the form $P(x) = x^k + c_{k-1}x^{k-1} + \dots + c_1x + c_0$, where c_0, c_1, \dots, c_{k-1} are non-negative integers, such that

$$P(a_n) = a_{n+1}a_{n+2} \cdots a_{n+k}$$

for every integer $n \geq 1$.



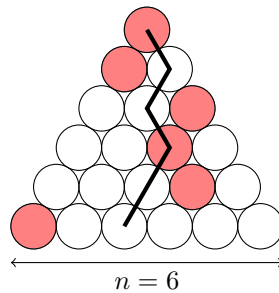
Sunday, 9. July 2023

Problem 4. Let $x_1, x_2, \dots, x_{2023}$ be pairwise different positive real numbers such that

$$a_n = \sqrt{(x_1 + x_2 + \dots + x_n) \left(\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n} \right)}$$

is an integer for every $n = 1, 2, \dots, 2023$. Prove that $a_{2023} \geq 3034$.

Problem 5. Let n be a positive integer. A *Japanese triangle* consists of $1 + 2 + \dots + n$ circles arranged in an equilateral triangular shape such that for each $i = 1, 2, \dots, n$, the i^{th} row contains exactly i circles, exactly one of which is coloured red. A *ninja path* in a Japanese triangle is a sequence of n circles obtained by starting in the top row, then repeatedly going from a circle to one of the two circles immediately below it and finishing in the bottom row. Here is an example of a Japanese triangle with $n = 6$, along with a ninja path in that triangle containing two red circles.



In terms of n , find the greatest k such that in each Japanese triangle there is a ninja path containing at least k red circles.

Problem 6. Let ABC be an equilateral triangle. Let A_1, B_1, C_1 be interior points of ABC such that $BA_1 = A_1C$, $CB_1 = B_1A$, $AC_1 = C_1B$, and

$$\angle BA_1C + \angle CB_1A + \angle AC_1B = 480^\circ.$$

Let BC_1 and CB_1 meet at A_2 , let CA_1 and AC_1 meet at B_2 , and let AB_1 and BA_1 meet at C_2 . Prove that if triangle $A_1B_1C_1$ is scalene, then the three circumcircles of triangles AA_1A_2 , BB_1B_2 and CC_1C_2 all pass through two common points.

(Note: a scalene triangle is one where no two sides have equal length.)

Mathematics

1.1 Algebra

Simplification of formulae by factorisation and expansion. Solving linear systems of equations. Solving equations and systems of equations leading to quadratic and biquadratic equations; selection of physically meaningful solutions. Summation of arithmetic and geometric series

1.2 Functions

Basic properties of trigonometric, inverse-trigonometric, exponential and logarithmic functions and polynomials.

This includes formulae regarding trigonometric functions of a sum of angles, Solving simple equations involving trigonometric, inverse-trigonometric, logarithmic and exponential functions.

1.3 Geometry and stereometry

Degrees and radians as alternative measures of angles.

Equality of alternate interior and exterior angles, equality of corresponding angles. Recognition of similar triangles. Areas of triangles, trapezoids, circles and ellipses; surface areas of spheres, cylinders and cones; volumes of spheres, cones, cylinders and prisms. Sine and cosine rules, property of inscribed and central angles, Thales' theorem, medians and the centroid of a triangle. Students are expected to be familiar with the properties of conic sections including circles, ellipses, parabolae and hyperbolae.

1.4 Vectors

Basic properties of vectorial sums, dot and cross products. Double cross product and scalar triple product. Geometrical

interpretation of a time derivative of a vector quantity.

1.5 Complex numbers

Summation, multiplication and division of complex numbers; separation of real and imaginary parts. Conversion between algebraic, trigonometric, and exponential representations of a complex number. Complex roots of quadratic equations and their physical interpretation.

1.6 Statistics

Calculation of probabilities as the ratio of the number of objects or event occurrence frequencies. Calculation of mean values, standard deviations, and standard deviation of group means.

1.7 Calculus

Finding derivatives of elementary functions, their sums, products, quotients, and nested functions. Integration as the inverse procedure to differentiation. Finding definite and indefinite integrals in simple cases: elementary functions, sums of functions, and using the substitution rule for a linearly dependent argument. Making definite integrals dimensionless by substitution. Geometric interpretation of derivatives and integrals. Finding constants of integration using initial conditions. Concept of gradient vectors (partial derivative formalism is not needed).

1.8 Approximate and numerical methods

Using linear and polynomial approximations based on Taylor series. Linearization of equations and expressions.

Perturbation method: calculation of corrections based on

unperturbed solutions. Finding approximate numerical solutions to equations using, e.g., Newton's method or bisection of intervals. Numerical integration using the trapezoidal rule or adding rectangles.